Model for the Rain Erosion of Fiber Reinforced Composites

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The behavior of both uncoated and coated fiber reinforced composite materials subjected to repeated impingements of liquid droplets was investigated. Two systems were studied: 1) uncoated fiber reinforced composites and 2) fiber reinforced composites covered by a single layer of homogeneous coating of arbitrary thickness. Employing the fatigue theorems established for the rain erosion of homogeneous materials, algebraic equations were derived which describe the incubation period, rate of mass removal, and the total mass loss. The results were compared to available experimental data and good agreement was found between the present analytical results and the data.

Nomenclature

a_1-a_2 = constants (dimensionless)
b = constant defined by Eq. (31) (dimensionless)
b_2 = knee in the fatigue curve (dimensionless)
C = speed of sound (or equivalent wave speed) (fps)
d = diameter of the droplet (ft)
E = Young's modulus (lbf/ft^2)
E_{11} = longitudinal Young's modulus (lbf/ft^2)
E_{22} = transverse Young's modulus (lbf/ft^2)
f = number of stress cycles [Eq. (7)]
F = force (lbf)
G = shear modulus (lbf/ft^2)
G_{12} = longitudinal shear modulus (lbf/ft^2)
G_{23} = transverse shear modulus (lbf/ft^2)
h = thickness of coat (ft)
I = rain intensity (fps)
k_e = maximum number of stress wave reflections in the coating (dimensionless)
$\bar{k}$ = number of stress wave reflections in the coating during the time of impact (dimensionless)
m = mass eroded per unit area (lbm/ft^2)
m^* = dimensionless mass loss defined by Eq. (50)
n = number of drops impinging per unit area (number/ft^2)
n^* = number of drops impinging per site, see Eq. (34) (dimensionless)
N = fatigue life, see Eq. (31) (dimensionless)
P = stress (lbf/ft^2)
q = drop density (number/ft^3)
r = distance (ft)
S = parameter defined by Eq. (33) (lbf/ft^2)
S_e = parameter defined by Eq. (40) (lbf/ft^2)
t = time (sec)
t_L = the duration of impact (sec)
V = velocity of impact (fps)
V_t = terminal velocity of a rain droplet (fps)
V_f = volume fraction of fibers in composite materials (dimensionless)
V_m = volume fraction of matrix in composite materials (dimensionless)
W = weight loss due to erosion (lbf)
Z = dynamic impedance (lbm/ft^2-sec)
α = rate of mass loss (lbm/impact)
α^* = dimensionless rate of mass loss, see Eq. (47)
γ = parameter defined by Eq. (43) (dimensionless)
ϕ = the angle between axis and fiber's orientation (rad)
υ = Poisson's ratio (dimensionless)
υ_{12} = longitudinal Poisson's ratio (dimensionless)
υ_{21} = transverse Poisson's ratio (dimensionless)
μ_1 = $\cos\phi$, see Eq. (20) (dimensionless)
μ_2 = $\sin\phi$, see Eq. (20) (dimensionless)
ρ = density (lbm/ft^3)
θ = angle of impact (radians)
σ = stress (lbf/ft^2)
$\bar{\sigma}_o$ = mean stress at the liquid-coating interface (lbf/ft^2)
σ_I = endurance limit (lbf/ft^2)
σ_u = ultimate tensile strength (lbf/ft^2)
ψ = parameter defined by Eq. (42)

Subscripts

c = coating
f = filament
i = end of incubation period
m = matrix
L = liquid
s = solid
sc = coat-substrate interface
Lc = liquid-coating interface

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I. Introduction

NONMETALLIC components constitute an ever increasing portion of modern high speed aircraft, owing to their favorable performance characteristics including high strength to weight ratio and good magnetic and optical properties. Unfortunately, such components are susceptible to heavy damage when subjected to repeated impingements of liquid droplets. To utilize the full potential of nonmetallic components the damage caused to them by rain erosion must be understood.

The behavior of homogeneous materials (both metallic and nonmetallic) has been investigated extensively in the past both experimentally and analytically. However, most previous studies on reinforced composites are experimental in nature. The objective of this investigation was to develop analytical expressions which are consistent with experimental observations and which predict quantitatively the "erosion" of fiber reinforced materials under previously untested conditions. The model presented here describes a) the "incubation period," i.e., the time elapsed before the mass loss becomes appreciable, and b) the mass loss past the incubation period.

The model used in this study is based on fatigue concepts and is designed along the lines developed previously for homogeneous materials.[1,2] Here, the model is applied to both coated and uncoated fiber reinforced composites. Study of uncoated composites is important for the general understanding of the rain erosion behavior of such materials. The analysis of coated composities, however, is of greater practical significance, since most uncoated composites have relatively poor resistance to erosion and must be coated for erosion protection.

II. The Problem

The problem investigated is the following. Spherical liquid droplets of constant diameter d impinge repeatedly upon a semi-infinite material (Fig. 1). Two cases are considered: 1) the material is a fiber reinforced composite composed of unidirectional filaments embedded in a matrix. The material is taken to be semi-infinite normal to the plane of the surface (x direction, Fig. 1). 2) The material is a fiber reinforced composite as described in point (1), but is covered by a homogeneous coating of thickness h. In the analysis it is assumed that: a) the composites are macroscopically homogeneous; b) the fiber filaments are randomly distributed; c) there is no fiber contiguity; d) locally both the matrix and the filament are homogeneous and isotropic; e) the filaments are parallel to the surface; and f) there is a perfect bond between the matrix and the filaments and, in case of coated composites, between the coating and the substrate (i.e., at the interfaces the stresses and the displacements are continuous). The reinforced composite, the coating, and the droplets are characterized by the properties shown in Fig. 1 and Table 1.

The angle of incidence of the droplets θ, and the velocity of impact V are taken to be constant. The spatial distribution of the droplets is considered to be uniform. The number of droplets impinging on unit area in time t may be written as[1]

$$n=(6/\pi)\,[V\cos\theta/V_t d^3]\,(t)(I) \tag{1}$$

where I is the rain intensity and V_t the terminal velocity of the droplet. The impingement rate is assumed to be sufficiently low so that all the effects produced by the impact of one droplet diminish before the impact of the next droplet.

The pressure at the liquid-solid interface is taken to be constant and is approximated by the water hammer pressure[3]

$$P=\rho_L C_L V\cos\theta/[(\rho_L C_L/\rho_s C_s)+1] \tag{2}$$

where ρ and C are the density and the speed of sound. The subscripts L and s refer to the liquid and solid, respectively. For a homogeneous materials ρ_s and C_s are the density and speed of sound of the material. For a fiber reinforced composite ρ_s and C_s may be expressed as

$$\rho_s=\rho_f V_f+\rho_m V_m \qquad C_s=[E_{22}/\rho_s]^{1/2} \tag{3}$$

where the subscripts f and m refer to the filament and the matrix, respectively. V is the volume fraction, so that the total volume of the composite is

$$V_s=V_f+V_m=1 \tag{4}$$

E_{22} is the equivalent Young's modulus in the direction normal to the fibers [see Eq. (17)].

For the purposes of the present analysis, Eqs. (2) and (3) represent the pressure with sufficient accuracy. The duration of this pressure is approximated by

$$t_L=(2d/C_L) \tag{5}$$

The forces, created by the impingements of the droplets, damage the material. This damage manifests itself in different ways, by cracks and pits, and by weight loss of the material. Here, we consider the weight loss to represent material damage, and describe the weight loss of the material as a function of time (Fig. 2a). To facilitate the analysis we replace the total weight loss by mass loss per unit area m, and the time by the number of droplets impinging per unit area n (Fig. 2 b). The data is then approximated by two straight lines, as shown in Fig. 2b. Accordingly, the mass loss is given by the expressions

$$m=0 \qquad 0<n<n_i \tag{6a}$$

$$m=\alpha(n-n_i) \qquad n_i<n<_f \tag{6b}$$

In Eqs. (6a) and (6b) n_i is the incubation period, a period during which the mass loss is insignificant, α is the rate of mass loss subsequent to the incubation period, and n_f is the limit beyond which the data deviates from the straight line relationship (in most practical situations, the usefulness of the material does not extend beyond n_f). Hence, the mass loss—and the erosion damage—can be evaluated once the parameters n_i, α, and n_f are known. Therefore, the problem is to determine these parameters.

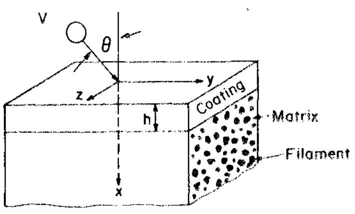

Fig. 1 Droplet impingement on a coat-substrate system: diameter, d; density, ρ_L; and speed of sound, C_L.

Table 1 Droplet impingement on a coat-substrate system

	Coating	Substrate Matrix	Substrate Filament
Density	ρ_c	ρ_m	
Speed of sound	C_c	C_m	C_f
Modulus of elasticity	E_c	E_m	E_f
Poisson ratio	ν_c	ν_m	ν_f
Ultimate tensile strength	$(\sigma_u)_c$	$(\sigma_u)_m$	$(\sigma_u)_f$
Endurance limit	$(\sigma_I)_c$	$(\sigma_I)_m$	$(\sigma_I)_f$
Volume fraction		V_m	V_f
Thickness	h		

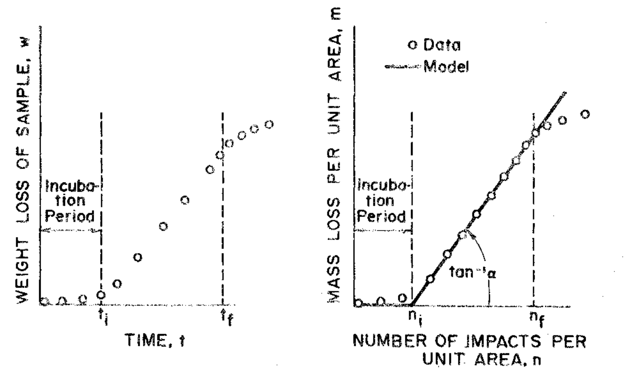

Fig. 2 a) Schematic of the experimental results; b) The solution model.

III. Incubation Period of Uncoated Fiber Reinforced Composites

In their previous investigations of coated and uncoated homogeneous materials, Springer and his coworkers[1,2] found that the incubation period can be established by applying fatigue theorems to the rain erosion problem. This approach is followed here and, in the following, fatigue concepts are used to determine the incubation period for fiber reinforced composites.

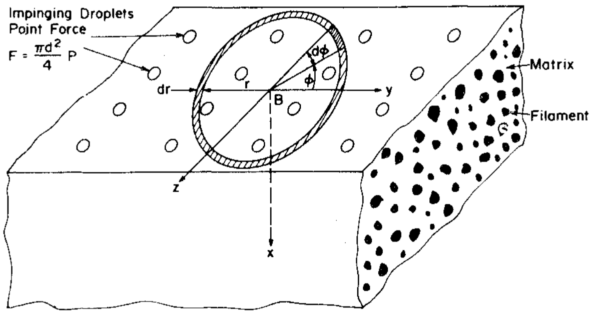

Fig. 3 Force distribution on the surface.

The starting point of the analysis is Miner's rule, which states that the failure of bars undergoing repeated torsion or bending obeys the expression[4]

$$(f_1/N_1) + (f_2/N_2) + \ldots + (f_q/N_q) = a_1 \tag{7}$$

where $f_1, f_2 \ldots f_q$ represent the number of cycles the specimen is subjected to specified overstress levels $\sigma_1, \sigma_2 \ldots \sigma_q$, and $N_1, N_2 \ldots N_q$ represent the life (in cycles) at these overstress levels, as given by the fatigue (σ_e vs N) curve.[2] a_1 is a constant. The value of a_1 is not necessarily unity.[5] For example, in torsion and bending tests it generally varies between 0.6 and 2.2.

Let us now consider a point on the surface of the material as shown in Fig. 3. Each droplet impinging on the surface creates a stress at point B. For a homogeneous, isotropic material this stress may be written as[6]

$$\sigma(r) = [F(1-2\nu)/2\pi r^2] \tag{8}$$

where F is the force of impact at a distance r from the point, and ν is the Poisson ratio. For composite materials a similar expression for the stress has not been established. Therefore, for the quasihomogeneous material under consideration here (see Sec. II), we shall approximate the stress by

$$\sigma(r,\phi) = (F[1-2\nu(\phi)]/2\pi r^2) \tag{9}$$

Now, σ is a function of both the distance of the point of impact r, and the orientation of the direction of the impact with respect to the direction of the fibers ϕ. The force is taken to be a point force, i.e.

$$F = (\pi d^2/4)P \tag{10}$$

where P is the water hammer pressure given by Eq. (2). ν_ϕ is the Poisson ratio for the composite in the ϕ direction[7]

$$\nu_\phi = \left[\frac{\nu_{12}}{E_{11}} - \left(\frac{1+2\nu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}} \right) \mu_1^2 \mu_2^2 \right] E_\phi \tag{11}$$

$$E_\phi = \left[\frac{\mu_1^4}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right) \mu_1^2 \mu_2^2 + \frac{\mu_2^4}{E_{22}} \right]^{-1} \tag{12}$$

and G_ϕ is the shear modulus of the composite in the ϕ direction[7]

$$G_\phi = \left[\frac{1}{G_{12}} + 4\left(\frac{1+2\nu_{12}}{E_{11}} + \frac{1}{E_{22}} - \frac{1}{G_{12}} \right) \mu_1^2 \mu_2^2 \right]^{-1} \tag{13}$$

In the longitudinal direction (i.e., in the direction parallel to the fibers) the Young's and shear moduli and the Poisson ratio may be written as[7-9]

$$E_{11} = E_f V_f + E_m V_m \tag{14}$$

$$G_{12} = \frac{(G_f + G_m)\, G_m V_m + 2G_f G_m V_f}{(G_f + G_m)\, V_m + 2G_m V_f} \tag{15}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \tag{16}$$

In the transverse direction (i.e., normal to the fibers) the moduli are

$$E_{22} = 4(\bar{Z}\bar{X} - \bar{Y}^2) G_{23} / [(\bar{X} + G_{23})\bar{Z} - \bar{Y}^2] \tag{17}$$

$$G_{23} = G_m \times \frac{2V_f G_f (X_m + G_m) + 2V_m G_f G_m + V_m X_m (G_f + G_m)}{2V_f G_m (X_m + G_m) + 2V_m G_f G_m + V_m X_m (G_f + G_m)} \tag{18}$$

$$\nu_{21} = \nu_{12}(E_{22}/E_{11}) \tag{19}$$

μ_1 and μ_2 are defined as

$$\mu_1 = \cos\phi \qquad \mu_2 = \sin\phi \tag{20}$$

In Eq. (17) $\bar{X}$, $\bar{Y}$, and $\bar{Z}$ are defined as

$$\bar{X} \equiv \frac{X_m (X_f + G_m) V_m + X_f (X_m + G_m) V_f}{(X_f + G_m) V_m + (X_m + G_m) V_f} \tag{21}$$

$$\bar{Y} \equiv V_f Y_f + V_m Y_m + [(Y_f - Y_m)/(X_f - X_m)] \times (\bar{X} - V_f X_f - V_m X_m) \tag{22}$$

$$\bar{Z} \equiv 2V_f(1-\nu_f)X_f + 2V_m(1-\nu_m)X_m + [(Y_f - Y_m)/(X_f - X_m)]^2 \, (\bar{X} - V_f X_f - V_m X_m) \tag{23}$$

where

$$X_{f,m} \equiv \frac{E_{f,m}}{2(1+\nu_{f,m})\,(1-2\nu_{f,m})} \tag{24a}$$

$$Y_{f,m} \equiv \frac{\nu_{f,m} E_{f,m}}{(1+\nu_{f,m})\,(1-2\nu_{f,m})} \tag{24b}$$

During the incubation period the total number of impacts on an $rdrd\phi$ element located at r is (Fig. 3)

$$f(r,\phi) = n_i r \, dr d\phi \tag{25}$$

Accordingly, we write Miner's rule Eq. (7) in the form

$$\frac{f(r_1,\phi)}{N_1} + \frac{f(r_2,\phi)}{N_2} + \ldots + \frac{f(r_q,\phi)}{N_q} = a_1 \tag{26}$$

Since r varies continuously from zero to infinite and ϕ from zero to 2π, Eq. (26) may be written as

$$\int_0^\infty \int_0^{2\pi} \left(\frac{n_i r d\phi dr}{N} \right) = a_1 \tag{27}$$

Equation (9) may be rearranged in the form

$$rdr = -(1/2\pi)\, [F(1-2\nu)/2\sigma^2] d\sigma \tag{28}$$

Substituting Eq. (28) into Eq. (27), and using the relationship given by Eq. (10) we obtain

$$-\int_0^{2\pi}\int_{\sigma_u}^{\sigma_I} n_i \left[n_i [\, \frac{P\pi d^2}{4} \cdot \frac{1}{2\pi} \quad \frac{(1-2\nu)}{2\sigma^2} \right] d\sigma d\phi / N = a_1 \tag{29}$$

The lower and upper limits of the first integral have been changed to the ultimate tensile strength and the endurance limit of the material, respectively. We shall assume that failure first occurs in the matrix, and approximate σ_u and σ_I by

$$\sigma_u \cong \sigma_{u_m} (E\phi/E_m) \qquad \sigma_I \cong \sigma_{I_m} (E\phi/E_m) \tag{30}$$

where E_ϕ is given by Eq. (12). The integrations in Eq. (29) may be performed once the fatigue life N is known as a function of the stress σ. N may be expressed as[1]

$$N=(\sigma_u/\sigma)^b \tag{31a}$$

where

$$b \equiv b_2/[\log_{10}(\sigma_u/\sigma_I)] = b_2/[\log_{10}(\sigma_{u_m}/\sigma_{I_m})] \tag{31b}$$

b_2 is a constant, such that 10^{b_2} corresponds to the "knee" in the fatigue curve. Substituting Eq. (31a) into Eq. (29), replacing σ by the equivalent dynamic stress[2] $\sigma_e=\sigma_u(\sigma/2)/(\sigma_u-\sigma/2)$, and integrating we obtain

$$\frac{\pi d^2}{4} P n_i \frac{1}{4(b-1)} \frac{E_m}{\sigma_{um}} \left[1-\left[\frac{\sigma_{Im}}{\sigma_{um}}\right]^{b-1}\right]$$
$$\times\left\{3/8\left[\frac{1}{E_{11}}+\frac{1}{E_{22}}\right]+1/8\left[\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_{11}}\right]\right.$$
$$\left.-\frac{2\nu_{12}}{E_{11}}+1/4\left[\frac{1+2\nu_{12}}{E_{11}}+\frac{1}{E_{22}}-\frac{1}{G_{12}}\right]\right\}=a_1 \tag{32}$$

Introducing the definitions

$$S=\frac{4(b-1)}{1-(\sigma_{Im}/\sigma_{um})^{b-1}} \frac{\sigma_{um}}{E_m}\left\{3/8\left[\frac{1}{E_{11}}\right.\right.$$
$$\left.+\frac{1}{E_{22}}\right]+1/8\left[\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_{11}}\right]$$
$$\left.-\frac{2\nu_{12}}{E_{11}}+1/4\left[\frac{1+2\nu_{12}}{E_{11}}+\frac{1}{E_{22}}-\frac{1}{G_{12}}\right]\right\}^{-1} \tag{33}$$

$$n_i^*=(\pi d^2/4)n_i \tag{34}$$

Eq. (32) becomes

$$n_i^*=a_1(S/P) \tag{35}$$

It is noted now that Eq. (35) is similar to the expression obtained in Ref. 1 for homogeneous materials. As in the case of homogeneous materials, S represents the "strength" of the material, while P is the stress produced at the surface. Naturally, the expression for S for reinforced composites [Eq. (33)] is different from the value of S for homogeneous materials. However, as one would expect in the limits: a) when there is only one constituent present; i.e., when

$$V_m=0 \qquad V_f=1 \qquad \text{or } V_f=0 \qquad V_m=1 \tag{36}$$

or b) when the fiber and matrix materials are identical

$$E_m=E_f \qquad \nu_f=\nu_m \qquad G_{12}=G_{23}=G_m=G_f \tag{37}$$

Eq. (33) reduces to the same form as was obtained previously for homogeneous materials.

The foregoing analysis is based on fatigue properties of bars in pure torsion and bending. Consequently, a linear relationship cannot hold between n_i^* and S/P. To extend the range of applicability of Eq. (35), while retaining its major feature (namely the functional dependence of n_i^* on S/P), we write

$$n_i^*=a_1(S/P)^{a_2} \tag{38}$$

where a_1 and a_2 are constants. For homogeneous materials these constants were found to be[2] $a_1=7.1\times10^{-6}$ and $a_2=5.7$. The same values of these constants will be used here; i.e.,

$$n_i^*=7.1\times10^{-6}(S/P)^{5.7} \tag{39}$$

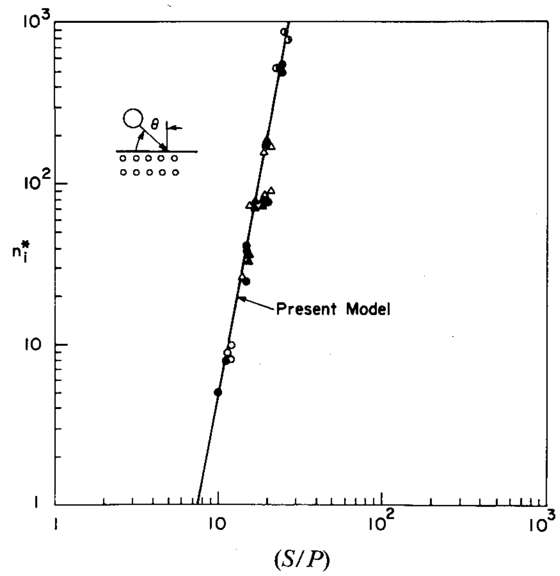

Fig. 4 Incubation period n_i^* vs (S/P). Droplet impingement on an uncoated composite substrate. Solid line: model Eq. (39). Symbols defined in Table 2.

The use of the previous constants insures that in the limits given by Eqs. (36) and (37) the foregoing expression yields the result appropriate for a homogeneous material. In other words, Eq. (39) together with Eq. (33) may be applied for all volume fractions of the filament from $V_f=0(V_m=1)$ to $V_f=1(V_m=0)$. The only limitation on the result is that the incubation period must be greater than zero. The conditions necessary for this limit are further discussed in Sec. VII.

The validity of the model was evaluated by comparing the previous analytical results to experimental data. This comparison, shown in Fig. 4, includes all existing data known to us for which the relevant material properties were available. As can be seen from Fig. 4 there is very good agreement between the present result and the data. This lends further confidence to the rain erosion model based on fatigue concepts.

IV. Incubation Period for Coated Fiber Reinforced Composites

The incubation period for a homogeneous coating on a homogeneous substrate was found by Springer, Yang and Larsen[2] to be

$$n_i^*=7.1\times10^{-6}\left[\frac{S}{\bar{\sigma}_o}\frac{1}{1+2\bar{k}\|\psi_{sc}\|}\right]^{5.7}$$
$$=7.1\times10^{-6}(S_e/\bar{\sigma}_0)^{5.7} \tag{40}$$

where

$$\bar{k}=k_e[1-\exp(-\gamma)] \tag{41}$$

and

$$\psi_{sc}\equiv(Z_s-Z_c)/(Z_s+Z_c)$$
$$\psi_{Lc}\equiv(Z_L-Z_c)/(Z_L+Z_c) \qquad Z\equiv\rho C \tag{42}$$

$$k_e=(1-\psi_{sc}\psi_{Lc})^{-1}$$
$$\gamma=(C_c/C_L)(d/h)\left[\frac{1+Z_L/Z_s}{1+Z_c/Z_s}\right]\left[\frac{2}{1+Z_L/Z_c}\right] \tag{43}$$

$$\bar{\sigma}_o=P\frac{1+\psi_{sc}}{1-\psi_{sc}\psi_{Lc}}$$
$$\times\left[1-\psi_{sc}\frac{1+\psi_{Lc}}{1+\psi_{sc}}\frac{1-\exp(-\gamma)}{\gamma}\right] \tag{44}$$

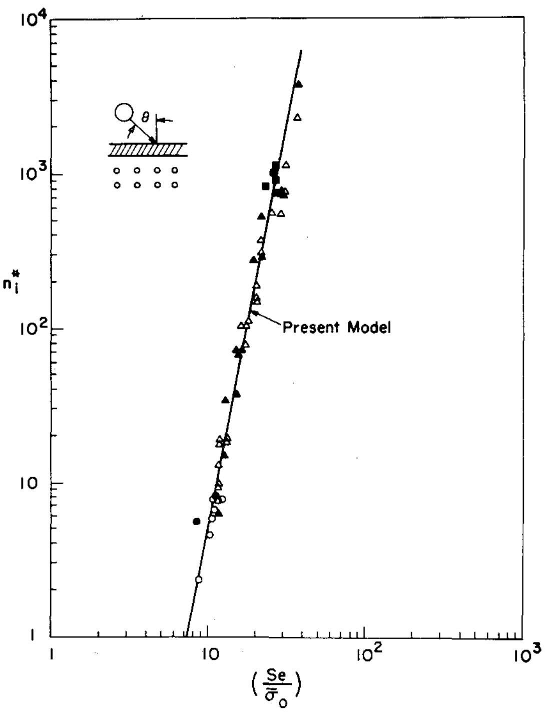

Fig. 5 Incubation period n_i^* vs $(S_e/\bar{\sigma}_0)$. Droplet impingement on a coated composite substrate. Solid line: model Eq. (40). Symbols defined in Table 3.

S is based on the properties of the coating. In the absence of the coating $\bar{\sigma}_o$ becomes P and the incubation period is[1]

$$n_i^* = 7.1 \times 10^{-6} (S/P)^{5.7} \tag{45}$$

The analysis leading to Eqs. (40-44) is valid even when the substrate is a fiber reinforced composite material, provided the fibers are randomly distributed and the composite can be taken to be quasihomogeneous. The incubation period can thus be calculated from Eq. (40). The properties of the substrate enter only through the impedance Z_s. In calculating Z_s, ρ_s and C_s given by Eq. (3) must be used. The calculated values of the incubation period are compared to available experimental data in Fig. 5. Again good agreement is evident between the results of the model and the data.

V. Rate of Mass Removal

The rate of mass removal for a homogeneous material with or without a coating may be calculated from the expression[1,2]

$$\alpha^* = 0.023 (1/n_i^*)^{0.7} \tag{46}$$

where α^* is defined as

$$\alpha^* \equiv (4\alpha/\pi\rho d^3) \tag{47}$$

The arguments leading to the previous results could be repeated, without any modification, for fiber reinforced composite materials. Since the analyses for homogeneous materials are presented in detail in Refs. 1 and 2, they are not repeated here. It suffices to say that this result is applicable to fiber reinforced composite materials (both with and without coating) as well as to homogeous materials. Naturally, the appropriate equation must be used in evaluating n_i^*. For a fiber reinforced composite material n_i^* must be calculated either from Eq. (39) (uncoated) or from Eq. (40) (coated).

In Eq. (47) ρ is the density of the material undergoing erosion. For an uncoated fiber reinforced composite material ρ is given by Eq. (3); for a coated material ρ is the density of the coating.

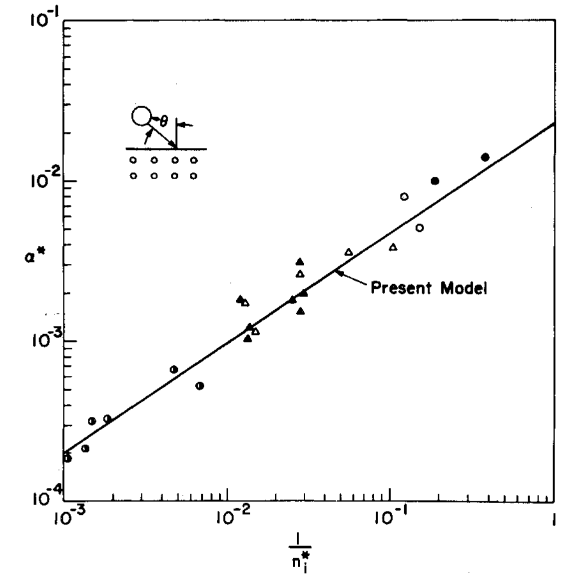

Fig. 6 Rate of erosion vs the inverse of the incubation period. Droplet impingement on an uncoated composite substrate. Solid line: model Eq. (46). Symbols defined in Table 2.

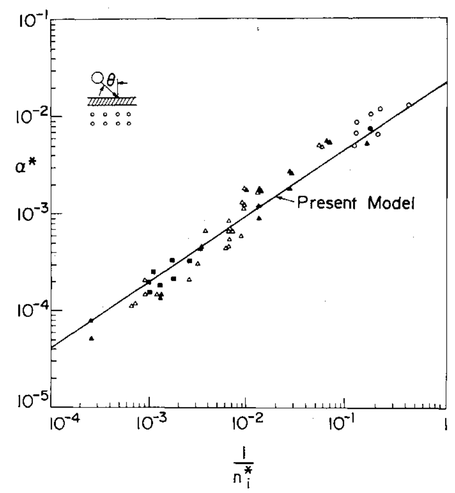

Fig. 7 Rate of erosion vs the inverse of the incubation period. Droplet impingement on a coated composite substrate. Solid line: model Eq. (46). Symbols defined in Table 3.

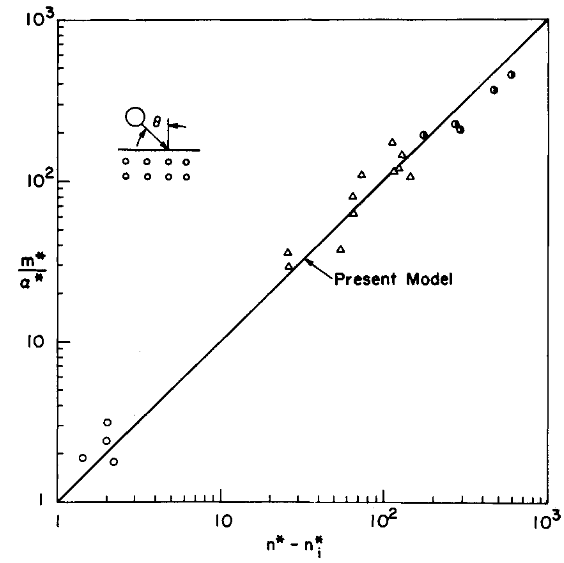

Fig. 8 Comparison of present model [solid line, Eq. (49)] with experimental results. Droplet impingement on an uncoated composite substrate. Symbols defined in Table 2.

Table 2 Description of data and symbols used in Figs. 4, 6, 8

Symbol	Investigators	Materials Fiber	Materials Matrix	Impact velocity fps	Rain intensity in./hr	Drop size mm	%Fiber
▲	Lapp et al.[10]	glass	polyester epoxy	731	1	1.9	65
Δ	Lapp et al.[11]	glass ceramic	polyester epoxy polyimide Teflon	731	1	1.9	65
○	Schmitt & Krabill[12]	glass	epoxy silicone polyester polyphenylene	1596 2360 2751 4246	2.5	1.9	69
		boron graphite	epoxy				55-65
●	Schmitt[14]	boron graphite	epoxy	731	1	1.8	55-65
◐	Schmitt[13]	glass	polyimide epoxy	731	1	1.8	65
		graphite boron	epoxy				55-65

Table 3 Description of data and symbols used in Figs. 5, 7, and 9

Symbol	Investigator	Coating	Substrate Fiber	Substrate Matrix	Coating thickness, mils	Impact velocity fps	Rain intensity in./hr	Drop size mm	% Fiber
■	Lapp et al.[15]	silicone	glass	polyester	10	731	1	1.9	65
		neoprene			1,4,4,7,8, 9,10,13,14, 21,22,27-33				
▲	Lapp et al.[10]	neoprene	glass	polyester	4-5,10	731	1	1.9	65
		polyurethane			2,4,5,11,23				
		polyethylene			7				
		neoprene	glass	epoxy	10				
		Teflon			2,5				
Δ	Lapp et al[11]	neoprene	glass	polyester	5,6,8,10-15	731	1	1.9	65
		polyurethane			20,5-10				
		polyurethane	glass	epoxy	10,14,15-20				
		Teflon	glass	epoxy	10,15				
		hypalon	glass	epoxy	9				
○	Schmitt & Krabill[12]	alumina	glass glass	epoxy polyimide	20,30,40 30	1593 2246 2426	2.5	19	69
		neoprene	glass	epoxy	10	1556 2220			
		polyurethane			5,10,15,20 30	1556 2220 2350			
		polyethylene	glass	Epon 828	30	1678 2216 2451			
		polyurethane	boron	epoxy	10	1593 2360			55-65
		nickel	glass	Epon 828	12,15	3169 4291 2216 2280	25	1.9	69
●	Schmitt[14]	polyurethane nickel	graphite boron	epoxy epoxy	8,12	731	1	18	55-65

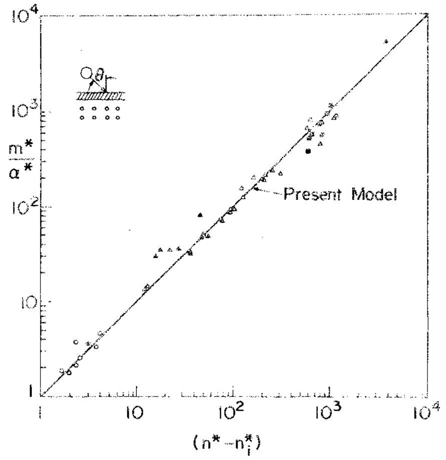

Fig. 9 Comparison of present model [solid line, Eq. (49)] with experimental results. Droplet impingement on a coated composite substrate. Symbols defined in Table 3.

The rates of mass removal, calculated from Eq. (46) are shown in Figs. 6 and 7. In these figures, the available experimental data are also given. The agreement is again good between the analytical results obtained from the present model and the data.

VI. Total Mass Loss

Equation (6b) is rewritten now in dimensionless form

$$m^* = \alpha^* (n^* - n_i^*) \tag{48}$$

or

$$m^*/\alpha^* = n^* - n_i^* \tag{49}$$

Here the dimensionless mass loss rate is defined as

$$m^* \equiv (m/\rho d) \tag{50}$$

ρ is again the density of the material undergoing erosion. Equation (50) is valid for both coated or uncoated materials but, of course, in calculating the mass loss rate the correct forms of n_i^* and α^* must be used.

Using Eq. (49), all the data can be correlated on a m^*/α^* vs $n^* - n_i^*$ plot. Such correlations are presented on Figs. 8 and 9. In Fig. 8, the analytical results are compared to the data for the case of uncoated composites. In Fig. 9, a similar comparison is given for coated composites. The agreement between the data and the theoretical line is very good, in fact, remarkable in view of the unavoidable errors inherent in many of the measurements. It must be emphasized that the theoretical lines in Figs. 8 and 9 are direct results of the calculations and are in no way "matched" to the data shown in these figures.

VII. Limits of Applicability of Model

The results presented in the foregoing sections are valid when the following two conditions are satisfied: 1) there is a finite incubation period, and 2) the mass loss varies linearly with the number of impacts n (i.e., with time t). The first of these conditions is met when the following inequality is satisfied

$$n_i^* > 1 \tag{51}$$

For a fiber reinforced composite without coating this condition may also be expressed as [see Eq. (39)]

$$(S/P) > 8 \tag{52}$$

For a fiber reinforced composite covered with a homogeneous coating a finite incubation period exists if [see Eq. (40)]

$$(S/\bar{\sigma}_o)[1 + 2\bar{k} \parallel \psi_{sc} \parallel]^{-1} > 8 \tag{53}$$

Equations (51-53) provide the lower limit of the applicability of the model. The upper limit beyond which the present model cannot be applied is determined by the second condition given. This limit was estimated by observing that up to about $n = 3n_i$ the data do not deviate significantly form the model. This condition may be expressed as

$$n < 3n_i \tag{54}$$

Conditions (51) and (54) are the only constraints imposed on the model. No further restrictions are placed on either the material or the impact velocity.

References

[1] Springer, G.S. and Baxi, C.B., "A Model for Rain Erosion of Homogeneous Materials," in *Erosion, Wear and Interfaces with Erosion*, ASTM STP 567, Dec. 1974, American Society for Testing and Materials, Philadelphia Pa., pp. 106-127.

[2] Springer, G.S., Yang, C.I., and Larsen, P.S., "Analysis of Rain Erosion of Coated Materials," *Journal of Composite Materials,* Vol. 8, July 1974, pp. 229-252.

[3] Engel, O.G., "Waterdrop Collisions with Solid Surfaces," *Journal of Research, National Bureau of Standards,* Vol. 54, May 1955, pp. 281-298.

[4] Miner, M.A., "Cumulative Damage in Fatigue," *Journal of Applied Mechanics,* Vol. 12, Sept. 1945, A159-A164.

[5] Juvinall, R.C., *Stress, Strain and Strength,* McGraw-Hill, New York, 1967.

[6] Timoshenko, S., *Theory of Elasticity,* McGraw-Hill, New York, 1934.

[7] Zecca, A.R. and Hay, D.R., "Elastic Properties of Metal-Matrix Composites," *Journal of Composite Materials,* Vol. 4, Oct. 1970, pp. 556-561.

[8] Hermans, J.J., "The Elastic Properties of Fiber-Reinforced Materials when the Fibers are Aligned," *Proceedings of the Koninkelijke Nederlandse Akademik Van Wetenschappen,* Amsterdam, Ser. B, Vol. 70, No. 1, 1967, pp. 1-11.

[9] Hill, R., "Theory of Mechanical Properties of Fiber-Strengthened Materials, I. Elastic Behavior," *Journal of the Mechanics and Physics of Solids,* Vol. 12, Sept. 1964, pp. 199-218.

[10] Lapp, R.R., Stutzman, R.H., and Wahl, N.E., "Summary Report on the Rain Erosion of Aircraft Materials at High Speed in Rain," WADC Tech. Rept. 53-185, Pt. 3, Sept. 1956, Wright-Patterson Air Force Base, Dayton, Ohio.

[11] Lapp, R.R., Thorpe, D.H., Stutzman, R.H., and Wahl, N.E., "The Study of Erosion of Aircraft Materials at High Speed in Rain," WADC Tech. Rept. 53-185, Pt. 4, May 1958, Wright-Patterson Air Force Base, Dayton, Ohio.

[12] Schmitt, G.F. and Krabill, A.H., "Velocity Erosion Rate Relationships of Materials in Rain at Supersonic Speeds," AFML-TR-70-44, Oct. 1970, Air Force Materials Lab., Wright-Patterson Air Force Base, Dayton, Ohio.

[13] Schmitt, G.F., "Materials Parameters that Govern the Rain Erosion Behavior of Polymeric Coatings and Composites at Subsonic Velocities," Tech. Rept. AFML-TR-71-197, Dec. 1971, Air Force Materials Lab., Wright-Patterson Air Force Base, Dayton, Ohio.

[14] Schmitt, G.F., Jr., "Rain-Erosion of Graphite and Boron-Fiber-Reinforced Epoxy Composite Materials," Technical Rept. AFML-TR-70-316, March 1971, Air Force Materials Lab., Wright-Patterson Air Force Base, Dayton, Ohio.

[15] Lapp, R.R., Stutzman, R.H. and Wahl, N.E., "A Study of the Rain Erosion of Plastic and Metals," WADC Tech. Rept. 53-185, Pt. 2, May 1955, Wright-Patterson Air Force Base, Dayton, Ohio.